USN | | | | | | | | | | | **06MAT41**

Fourth Semester B.E. Degree Examination, June 2012
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of statistical tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Employ Taylor's method to obtain approximate value of y at x = 0.1 and x = 0.2 for the differential equation $y' = x^2y - 1$, y(0) = 1 considering upto the fourth degree term. **(06 Marks)**

b. Using Runge-Kutta method of fourth order, solve : $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1 at x = 0.2, 0.4. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1 + y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, evaluate y(1.4) by Adams – Bashforth method. **(07 Marks)**

2 a. Obtain the Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Verify that $v = e^x (x \, \text{Sin}\, y + y \, \text{Cos}\, y)$ is harmonic. Find u such that f(z) = u + iv is an analytic function. Also find f(z). **(07 Marks)**

c. Find the region in the W-plane bounded by the lines x = 1, y = 1, x + y = 1 under the transformation $W = Z^2$. Indicate the region with sketches. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Find the Laurent's expansion for $f(z) = \frac{z^2}{(z-1)(z-3)}$ in the region i) $1 < |z| < 3$; ii) $|z-1| < 2$. **(07 Marks)**

c. Evaluate $\int_C \frac{\text{Sin}\pi z^2 + \text{Cos}\pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$, by Cauchy's residue theorem. **(07 Marks)**

4 a. Obtain the series solution of the equation $4xy'' + 2(1 - x)y' - y = 0$. **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$. **(07 Marks)**

c. Express $4x^3 - x^2 - 3x + 8$ interms of Legendre polynomial. **(07 Marks)**

PART – B

5 a. Fit a parabola of the form $y = a + bx + cx^2$ to the following data : **(06 Marks)**

x	0	1	2	3	4	5
y	1	3	7	13	21	31

b. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(07 Marks)

c. State and prove Baye's theorem. **(07 Marks)**

6 a. Find mean and standard deviation of the binomial distribution. **(06 Marks)**

b. The probability that an individual suffers a bad reaction from a certain injection is 0.001. Using Poisson distribution, determine the probability that out of 2000 individuals :

i) Exactly 3 and

ii) More than 2 will suffer a bad reaction. **(07 Marks)**

c. The weekly wages of workers in a company are normally distributed with mean of Rs.700/- and standard deviation of Rs.50. Find the probability that the weekly wage of a randomly chosen worker is i) between Rs.650 and Rs.750, and ii) more than Rs.750. **(07 Marks)**

7 a. The mean and standard deviation of marks scored by a sample of 100 students are 67.45 and 2.92. Find : i) 95% and ii) 99% confidence intervals for estimating the mean marks of the student population. **(06 Marks)**

b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.5}$ = 2.262 for 9 d.f). **(07 Marks)**

c. Explain the following terms :

i) Null hypothesis

ii) Confidence limits

iii) Type I and type II errors. **(07 Marks)**

8 a. A fair coin is tossed thrice. The random variables x and y are defined as follows :
x = 0 or 1 according as head or tail occurs on the first toss. y = number of heads.

i) Determine the marginal probability distribution of x and y.

ii) Determine the joint distribution of x and y.

iii) Determine E(x), E(y), E(xy).

iv) Determine σ_x, σ_y. **(06 Marks)**

b. Define Stochastic matrix. Show that the matrix P is a regular Stochastic matrix and also find its unique fixed probability vector.

$$P = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$$ **(07 Marks)**

c. A software engineer goes to his office everyday by motor bike or by car. He never goes by bike on two consecutive days. But if he goes by car on a day then he is equally likely to go by car or by bike the next day. Find the transition probability matrix of the Markov chain. If car is used on the first day of the week, find the probability that after 4 days

i) Bike is used

ii) Car is used. **(07 Marks)**

* * * * *